Introductory Exercise: E0.0

Stock Dynamics by hand

This little exercise will help you understand the principle of stock-driven models. It is **essential** that you understand this principle before you start programming this type of model in Python.

# Inflow-driven model – 4 years sharp lifetime

As in the first example, we assume a lifetime of exactly 4 years.

In this input-driven model, only the past inflows and the lifetime of each cohort are given.

How to modify the algorithm to solve the input-driven model? Is it still necessary to do a year-by-year calculation?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Annual stock**  (end of the year) | **Stock change**  (during the year) | **Inflow**  (during the year) | **Outflow**  (during the year) |
| **Year t** | **St**  **[tons]** | **dSt/dt**  **[tons/yr]** | **It**  **[tons/yr]** | **Ot**  **[tons/yr]** |
| t0=2000 | 0 | 0 | 0 | 0 |
| 2001 |  |  | 100 |  |
| 2002 |  |  | 200 |  |
| 2003 |  |  | 400 |  |
| 2004 |  |  | 800 |  |
| 2005 |  |  | 1600 |  |
| 2006 |  |  | 3000 |  |
| 2007 |  |  | 4000 |  |
| 2008 |  |  | 5000 |  |
| 2009 |  |  | 5500 |  |
| 2010 |  |  | 6000 |  |
| 2011 |  |  | 6000 |  |
| 2012 |  |  | 6000 |  |

Stock-driven models

A stock-driven model is a model in which the stock (measured at the end of the accounting year) and the lifetime of each cohort are given; both as independent variables. Subsequently, the stock change, the inflow, and the outflow are calculated (dependent variables).

**Task:** Calculate the missing data in the table below. Assume that the lifetime of the products in the stock is 4 years. While filling out the table, reflect on the following:

* What **operations** do you conduct to calculate the different dependent variables?
* In what **sequence** do you conduct the operations to fill out the entire table (e.g., column by column, or row by row, or some kind of iteration…)?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Annual stock**  (end of the year) | **Stock change**  (during the year) | **Inflow**  (during the year) | **Outflow**  (during the year) |
| **Year t** | **St**  **[tons]** | **dSt/dt**  **[tons/yr]** | **It**  **[tons/yr]** | **Ot**  **[tons/yr]** |
|  |  |  |  |  |
| t0=2000 | 0 | 0 | 0 | 0 |
| 2001 | 100 | 100 | 100 | 0 |
| 2002 | 200 |  |  |  |
| 2003 | 400 |  |  |  |
| 2004 | 800 |  |  |  |
| 2005 | 1600 |  |  |  |
| 2006 | 3000 |  |  |  |
| 2007 | 4000 |  |  |  |
| 2008 | 5000 |  |  |  |
| 2009 | 5500 |  |  |  |
| 2010 | 6000 |  |  |  |
| 2011 | 6000 |  |  |  |
| 2012 | 6000 |  |  |  |

**Next step: how to write these algorithms in Python, using a for loop?**

Solution and additional examples

# Stock-driven model – 4 years sharp lifetime

This table contains the solution to the first example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Annual stock**  (end of the year) | **Stock change**  (during the year) | **Inflow**  (during the year) | **Outflow**  (during the year) |
| **Year t** | **St**  **[tons]** | **dSt/dt**  **[tons/yr]** | **It**  **[tons/yr]** | **Ot**  **[tons/yr]** |
| t0=2000 | 0 | 0 | 0 | 0 |
| 2001 | 100 | 100 | 100 | 0 |
| 2002 | 200 | 100 | 100 | 0 |
| 2003 | 400 | 200 | 200 | 0 |
| 2004 | 800 | 400 | 400 | 0 |
| 2005 | 1600 | 800 | 800+100 = 900 | 100 |
| 2006 | 3000 | 1400 | 1400+100 = 1500 | 100 |
| 2007 | 4000 | 1000 | 1000+200 = 1200 | 200 |
| 2008 | 5000 | 1000 | 1000+400 = 1400 | 400 |
| 2009 | 5500 | 500 | 500+900 = 1400 | 900 |
| 2010 | 6000 | 500 | 500+1500=2000 | 1500 |
| 2011 | 6000 | 0 | 1200 | 1200 |
| 2012 | 6000 | 0 | 1400 | 1400 |

We can represent the results on a graph:

The algorithm used to find the solution can be described as follows:

We repeat these steps year by-year, for each year, starting in 2001:

1. Calculate the outflow from inflows from all previous years and the lifetime distribution. In this simple example, O(t) = I(t-4), but the general rule would be :

With the lifetime distribution for each cohort.

1. Calculate the stock change between the current year and the previous year:
2. Calculate the inflow:
3. Go back to step a. for the next year, until we reach 2012.

We notice that in a stock-driven model, the calculation must be done row by row (year by year). We cannot do it column by column, because in order to calculate the outflows for each cohort, we need to know all previous inflows.

# Stock-driven model – 3.5 years lifetime

In this new example, we now consider that for each cohort, half of the stock is retired after 3 years and the rest after 4 years. The average lifetime is then 3.5 years, and we can write the following equation for the outflows:

How does this new hypothesis on the lifetime affect the results?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Annual stock**  (end of the year) | **Stock change**  (during the year) | **Inflow**  (during the year) | **Outflow**  (during the year) |
| **Year t** | **St**  **[tons]** | **dSt/dt**  **[tons/yr]** | **It**  **[tons/yr]** | **Ot**  **[tons/yr]** |
| t0=2000 | 0 | 0 | 0 | 0 |
| 2001 | 100 |  |  |  |
| 2002 | 200 |  |  |  |
| 2003 | 400 |  |  |  |
| 2004 | 800 |  |  |  |
| 2005 | 1600 |  |  |  |
| 2006 | 3000 |  |  |  |
| 2007 | 4000 |  |  |  |
| 2008 | 5000 |  |  |  |
| 2009 | 5500 |  |  |  |
| 2010 | 6000 |  |  |  |
| 2011 | 6000 |  |  |  |
| 2012 | 6000 |  |  |  |